Minnesota State University, Mankato
43rd Annual High School Contest
April 20, 2016

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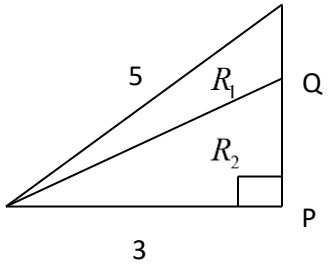
1. The value of $x = \overline{PQ}$ such that the area of region R_1 is one-fourth the area of region R_2 in the diagram below is



a. 3 b. 3/2 c. 16/5 d. 4 e. 15/4

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2. If the cost of producing 8 items is \$740, the cost of producing 13 items is \$1100, and the cost and number of items are linearly related, then the total fixed cost in dollars is
   a. 220 b. 164 c. 176 d. 210 e. 150

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3. The lengths of two sides of a triangle are 83 and 109. If P denotes the perimeter of the triangle, then
   a. $218 < P < 384$ b. $26 < P < 192$ c. $51 < P < 394$ d. $208 < P < 352$ e. $192 < P < 412$

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4. The product of the solutions to $2x^4 + 27 = 21x^2$ is
   a. 27 b. 27/2 c. 9 d. 9/2 e. n.o.t.

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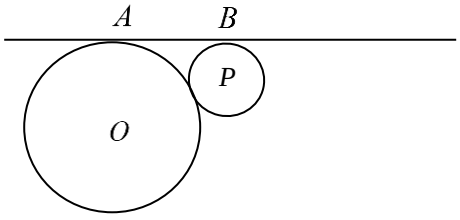
5. Circles centered at O, P below are externally tangent with radii 6 and 2 inches, respectively. If $\overleftrightarrow{AB}$ is tangent to both circles, then the area of trapezoid O, A, B, P in square inches is



a. 24 b. $10\sqrt{3}$ c. $16\sqrt{3}$ d. 18 e. 6π

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6. The expression $\dfrac{\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}}{\sqrt{\frac{3}{2}}-\sqrt{\frac{2}{3}}}$ can be simplified as

   a. $\sqrt{6}$ b. $2\sqrt{\frac{2}{3}}$ c. 5 d. $3\sqrt{\frac{3}{2}}$ e. 6

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7. A certain species of water lily doubles in area every 24 hours. If one water lily is put into a pond it takes 8 days to cover the pond. The number of water lilies of the same type it would take to cover the same pond in half the time is

   a. 2 b. 4 c. 5 d. 7 e. 16

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8. John has a slow bicycle tire leak. He hasn't bothered to fix it because it takes the tire 9 days to go flat after filling it. John acquired a 2nd puncture in the same tire, and now the tire will flatten in 3.6 days. The time in days it takes the 2nd puncture alone to flatten the tire is

   a. 4 b. 5.4 c. 6 d. 4.5 e. 12

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9. When the base of a ladder on level ground is 16 feet from the base of a vertical wall, 3 feet of the ladder projects beyond the top of the wall. When the base of the ladder is 9 feet from the base of the wall, 8 feet of the ladder projects beyond the top of the wall. The height of the wall in feet is

   a. 12 b. 16.5 c. 21. 5 d. 14 e. 18

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10. If $y=\ln\left(\dfrac{2x-1}{3x+5}\right)$, which of the following expresses x as a function of y?

    a. $x=\dfrac{5e^{y}-1}{3e^{y}-2}$ b. $x=\dfrac{5e^{y}+1}{2-3e^{y}}$ c. $x=\dfrac{2-3e^{y}}{5e^{y}-1}$ d. $x=\dfrac{1-5e^{y}}{2+3e^{y}}$ e. n.o.t.

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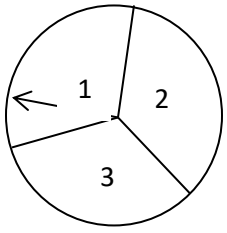
11. For the spinner below, regions 1, 2, and 3 have equal probability. The minimum number of spins n that will ensure a 90% probability or greater that the needle will stop in region 1 at least once in n spins is



    a. 5 b. 6 c. 7 d. 9 e. n.o.t.

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12. The range of $f(x)=e^{-(x^2-4x+7)^2}$ is

a. $(0,e^{-4}]$ b. $\left(0,e^{-9}\right]$ c. $\left(-\infty,e^{-9}\right)$ d. (e^{-2},∞) e. $[0,e^{-3})$

13. The sum of the solutions to $(2x-1)^2-3|2x-1|=10$ is

a. -4 b. -3 c. -2 d. -1 e. 1

14. The exact value of $\log_3(169)\log_{13}(243)$ is

a. 7 b. 1/4 c. 8 d. 1/7 e. n.o.t.

15. The length of the base of an isosceles triangle is $\sqrt{2}$. The medians to its legs meet at right angles. The area of the triangle is

a. $\sqrt{3}/2$ b. $3/2$ c. $\sqrt{2}/3$ d. $5/2$ e. 2

16. The amplitude, period, and phase shift, respectively, of $f(x)=-\sin(2x)+\sqrt{3}\cos(2x)$ are

a. $2,\pi,\pi/3$ b. $2,\pi,-\pi/6$ c. $1+\sqrt{3},\pi/2,-\pi/3$ d. $2,\pi,-\pi/3$ e. $2,2\pi,\pi/6$

17. If z is a complex number and $z^2=-21-20i,$ then the absolute value of $\mathrm{Re}(z)\,\mathrm{Im}(z)$ is

a. 5 b. 14 c. 12 d. 10 e. 42

18. It takes 10 man hours to make a table and 6 man hours to make a chair at the Wood Workers Shop. A minimum of 30 tables and 40 chairs must be made each day. The profit per table is \$150 and the profit per chair is \$65. If 720 man hours are available each day for making tables and chairs, and z is the sum of the numbers of tables and chairs that maximize profit, then z is

a. 70 b. 100 c. 85 d. 88 e. 92

19. If $x=a$ and $y=b$ is a solution to the system $\begin{cases} x+y+\sqrt{xy}=28 \\ x^2+y^2+xy=336 \end{cases}$, the smallest value of ab is

a. 64 b. 36 c. 32 d. 72 e. n.o.t.

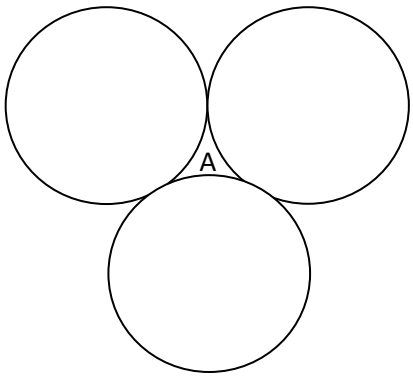
20. If $x=\sqrt{50-12\sqrt{14}}+\sqrt{50+12\sqrt{14}},$ then x is

a. a natural number b. the negative of a natural number c. a positive rational number that is not an integer d. a positive irrational number e. a negative irrational number

21. If $x = \sum_{n=0}^{\infty} [\tan^{-1}(n+1) - \tan^{-1}(n-1)]$, then x is

a. $\pi/4$ b. $\pi/2$ c. $3\pi/4$ d. π e. $5\pi/4$

22. Three mutually tangent circles of diameter 2 are shown in the figure below. The area of region A bounded by the three circles is



a. $\sqrt{3} - \frac{\pi}{2}$ b. $\frac{4\sqrt{3} - \pi}{3}$ c. $4\sqrt{3} - 2\pi$ d. $2\sqrt{6} - \pi$ e. n.o.t.

23. Four students are about to take a test containing 100 questions of equal difficulty. The probabilities are the first student will answer 60 questions correctly, the 2nd will answer 50 questions correctly, and the others will answer 40 and 25 questions correctly, resp. The probability that at least one of these four students will correctly answer the first question is

a. 91% b. 97% c. 85% d. 87.5% e. 43.75%

24. $p(x)$ and $q(x)$ are polynomials with positive integral coefficients such that $[q(x)]^2 = [p(x)]^2 + 2x^2 + 1$. Then the sum of the coefficients of $q(x)$ is

a. 0 b. 1 c. 2 d. 3 e. n.o.t.

25. If n, x are positive integers and $4^n = x^2 + 615$, then $n + x$ is

a. 55 b. 65 c. 59 d. 44 e. n.o.t.

26. The sum of the digits of the smallest positive solution ϑ measured in degrees to $\sin(20^\circ) + \sin(40^\circ) = \sin(\vartheta)$ is

a. 8 b. 7 c. 12 d. 10 e. n.o.t.

27. The 2007th digit in the decimal representation of $\frac{3}{7}+\frac{2006}{9999}$ is

a. 1 b. 2 c. 4 d. 7 e. 9

28. An airport is to be located equidistant to cities A,B, and C. If B is located 2 miles due east of A, C is located 2 miles south and 1 mile east of B, and the airport is located h miles east of A and k miles south of A, then h + k is

a. 11/4 b. 3 c. 5/2 d. 9/4 e. n.o.t.

29. If $a, b, \text{and } c$ are fixed positive real numbers not equal to 1, then the number of solutions x to $\log_a(x)\log_b(x)\log_c(x) = \log_a(x)\log_b(x) \mid \log_a(x)\log_c(x) \mid \log_b(x)\log_c(x)$ is

a. Infinite b. 4 c. 3 d. 2 e. 1

30. If the sum of two of the roots of $x^3 - ax^2 + bx + 4a = 0 \ \ (a \neq 0)$ is zero, then b is

a. -2 b. 0 c. 2 d. -4 e. 4

Tie Breaker

Name_____ **School Code**_____

Work the tie breaker problem on this page, tear off this sheet, and then hand it in along with your answer form. Partial credit will be given.

Tie Breaker:

1. Prove the Quadratic formula assuming $ax^2 + bx + c = 0 \ \ (a \neq 0)$
2. State and prove the formula for the lateral surface area for a right circular cone of radius r and height h.