

Minnesota State University, Mankato
44th Annual High School Mathematics Contest
April 12, 2017

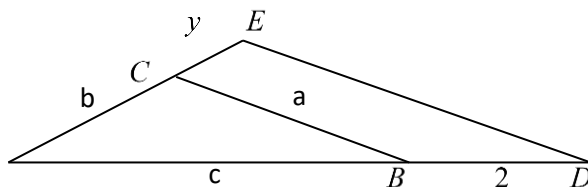
1. A 25 ft. ladder is placed against a vertical wall of a building. The foot of the ladder rests on the floor and is 7 ft. from the wall. If the top of the ladder slips 4 ft., then the amount the foot of the ladder slips in feet is

a. 9 b. 15 c. 5 d. 8 e. 4

2. The number of circular pipes with an inside diameter of 1 inch that will carry the same amount of water as a pipe with an inside diameter of 6 inches is

a. 9 b. 6 c. 12 d. 36 e. 18

3. The value of $y = CE$ in the diagram below such that $\triangle ABC$ is similar to $\triangle ADE$ is



a. 2 b. $\frac{2b}{c}$ c. $\frac{2a}{b}$ d. $\frac{b}{2c}$ e. $\frac{a}{2b}$

4. Two trains, one 500 ft. long and the other 400 ft. long, on parallel tracks, can pass each other in 9 sec. when moving in opposite directions. When moving in the same direction, the faster train can pass the slower train in 18 sec. The speed of the faster train in ft. per sec. is

a. 75 b. 50 c. 60 d. 90 e. 100

5. The average mark on an exam was 60%, and the average marks for those passing and failing the exam were 75% and 55%, respectively. The percentage of students passing the exam was

a. 50 b. 40 c. 35 d. 20 e. 25

6. The value of n such that $2^{10} + 2^{10} + 2^{10} + 2^{10} + 2^{10} + 2^{10} + 2^{10} + 2^{10} = 64^n$ is

a. $\frac{13}{6}$ b. $\frac{9}{4}$ c. $\frac{12}{5}$ d. $\frac{14}{5}$ e. $\frac{15}{7}$

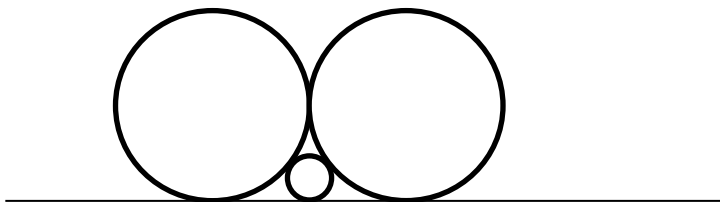
7. The sum of two numbers is 10 and their product is 20. The sum of their reciprocals is

- a. $\frac{1}{10}$ b. $\frac{1}{2}$ c. 1 d. 2 e. 4
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8. At a dance party a group of boys and girls exchange dances as follow: one boy dances with 5 girls, a second boy dances with 6 girls, and so on, and the last boy dances with all the girls. If b and g denote the number of boys and girls, respectively, then

- a. $b = g$ b. $b = \frac{1}{g}$ c. $b = g - 4$ d. $b = g - 5$ e. n.o.t.
-

9. Two circles, each of positive radius r inches, are placed so they are tangent to each other and are on a straight line as below. A smaller circle of positive radius x inches is nestled between them so that it is tangent to the circles and the line. The ratio of r to x is



- a. 5 b. 3.5 c. 4 d. 3 e. 2.5
-

10. A bathtub fills in a minimum of 12 minutes with the cold water on full tap. It fills in a minimum of 9 minutes with the hot water on full tap. The bathtub empties in a minimum of 6 minutes with the drain plug removed. Assuming the bathtub can hold more than 0 gallons, then the minimum amount of time in minutes it takes the bathtub to fill with both hot and cold water on full tap and the drain plug removed is

- a. 30 b. 45 c. 42 d. 36 e. 18
-

11. The minute and hour hand of a clock point in the same directions at a time between 8 am and 9 am. The number of minutes after 8 am at this time rounded to 2 decimal places is

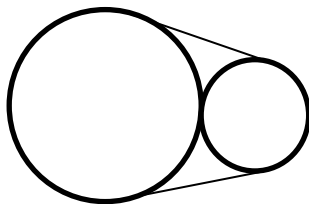
- a. 43.60 b. 44.23 c. 44.17 d. 43.67 e. 43.64
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12. A rectangle of perimeter 128 inches can be divided into three identical squares as below. Its area in square inches is



- a. 405 b. 750 c. 868 d. 768 e. 529
-
13. A cylindrical oil tank, lying horizontally, has an interior length of 10 ft. and an interior diameter of 6 ft. If the surface level of the oil it contains is 40 sq. ft., and the tank is at least half-full, then the depth of the oil in feet is
- a. 4 b. $\frac{10}{3}$ c. $3 + \frac{1}{\sqrt{5}}$ d. $3 + \sqrt{5}$ e. $3 + \sqrt{2}$
-
14. A stone is dropped into a well and the report of the stone striking the bottom is heard 7.7 seconds later. If the stone falls $16t^2$ ft. in t seconds and the speed of sound is 1,120 ft. per sec., then the depth of the well in feet is
- a. 784 b. 342 c. 1568 d. 156.8 e. 642
-
15. In a group of 100 tourists, 15 knew neither French nor Italian, 65 knew French and 77 knew Italian. The sum of the digits of the number of tourists that knew Italian but not French is
- a. 2 b. 3 c. 8 d. 6 e. 7
-
16. A small school has 100 students and rooms A, B, and C. After the 1st period, half the students in room A move to room B, $\frac{1}{5}$ of the students in room B move to room C, and $\frac{1}{3}$ of the students in room C move to room A, and these are the only changes. If the number of students in each room does not change, then the number of students in room A is
- a. 50 b. 40 c. 30 d. 20 e. 10
-
17. The maximum value of k for which $x^2 - 6x + k$ and $x^2 - 10x + 2k$ have a common zero is
- a. 0 b. 1 c. 4 d. 6 e. n.o.t.
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18. The length of the shortest belt in inches that will go around two tangent circles of diameters 6 inches and 18 inches as indicated below is



- a. $12\sqrt{3} + 16\pi$ b. $12\sqrt{3} + 7\pi$ c. $12 + 15\pi$ d. $12\sqrt{3} + 14\pi$ e. $32\frac{\sqrt{3}}{3}\pi$
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19. A boat has a speed of 15 mph in still water. In a current of 5 mph it travels a certain distance downstream and returns. Assuming all speeds are expressed in mph, the ratio of the average speed of the round trip to the average speed of the corresponding trip in still water is

- a. $\frac{8}{7}$ b. $\frac{8}{9}$ c. $\frac{7}{8}$ d. $\frac{4}{5}$ e. cannot be determined by anyone
-

20. The number of positive integers that leave a remainder of 24, as defined by the Division Algorithm, when divided into 4049 is

- a. 8 b. 7 c. 6 d. 5 e. 11
-

21. Let n be a positive integer. If the remainders of 77 and 121 are 7 and 9, respectively, when divided by n , then the sum of the base 10 digits of n are

- a. 7 b. 6 c. 5 d. 2 e. 1
-

22. Six people line up to use a drink machine that accepts only nickels and dimes in which each drink costs 5 cents. If 3 of the 6 people only have a nickel each, 3 only have a dime each, and the drink machine is initially out of change, then the probability, in percent, the six people line up randomly in such a way they each can purchase a drink is

- a. 15 b. 18 c. 20 d. 25 e. 5
-

23. Two players are dealt n cards. Assume the players can be dealt an unequal number of cards and each player must receive at least 1 card. The number of different hands possible is

- a. 2^n b. 2^{n-1} c. $2^n n!$ d. $2(2^{n-1} - 1)$ e. 2^{n-2}

24. If $(6^{30} + 6^{-30})(6^{30} - 6^{-30}) = 3^A 8^B - 3^{-A} 8^{-B}$, and A, B are real numbers, then B is

- a. 10 b. 20 c. 30 d. 40 e. cannot be determined by anyone
-

25. The number of real ordered pairs (x, y) at which the system $x = \sin(2\pi t)$ and $y = 2t - t^2$, for $t \in [0, 4]$, intersects itself is

- a. 0 b. 1 c. 2 d. 4 e. 8
-

26. If $\left(a + \frac{1}{a}\right)^3 = 3$, then $a^3 + \frac{1}{a^3}$ equals

- a. $\frac{10\sqrt{3}}{3}$ b. $-3\sqrt{3}$ c. $3(1 - 3^{\frac{1}{3}})$ d. $3(3^{\frac{1}{3}} - 1)$ e. $3\sqrt{3}$
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27. A boy wishes to climb a ladder that has 500 rungs (steps). He may go up the ladder only in positive integer multiples of 300 steps or down the ladder in only positive integer multiples of 198 steps, but he must remain on the ladder, once on the ladder. Assuming the rungs are enumerated from bottom to top from 1 to 500 and he starts with both feet on the ground, then the rung with the highest number he can step on is

- a. 300 b. 402 c. 432 d. 492 e. 498
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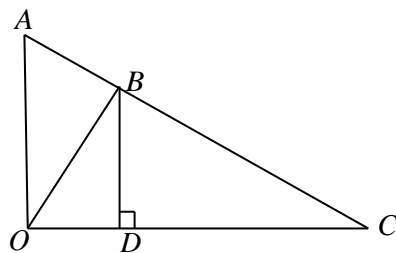
28. The smallest positive real number p such that $x > p$ implies $\sqrt{x^2 + x} - x$ differs from $\frac{1}{2}$ by less than .02 is

- a. 6.24 b. 5.74 c. 12.24 d. 5.7 e. 5.76
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29. A man born between 1800 and 1850 was x years old in the year x^2 . The sum of the digits of the year he was born is

- a. 15 b. 22 c. 16 d. 18 e. 17
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30. Given the right triangle AOC below, if $\triangle OBA$ is similar to $\triangle CBO$, $OA=3$, and $OC=6$, then BD is



- a. $\frac{6}{5}$ b. $\frac{12}{5}$ c. 2 d. $\frac{11}{5}$ e. $\frac{9}{5}$
-

Tie Breaker

Name _____

School Code _____

Work the tie breaker problem on this page, tear off this sheet, and then hand it in along with your answer form. Partial credit will be given.

Tie Breaker:

Prove that if m is a positive integer, then the equation $x^2 + y^2 = mx + my + m + 1 - 2xy$ has exactly m solutions (x, y) in which x, y are positive integers. The quality and completeness of the solution will be taken into consideration.