

**Minnesota State University, Mankato**  
**45<sup>th</sup> Annual High School Mathematics Contest**  
**April 25, 2018**

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1. The rectangle  $R$  below is divided into sub-rectangles  $A, B, C,$  and  $D$ . If the areas of  $A, B,$  and  $C$  are 30, 45, and 63 square units, respectively, then the area of  $R$  in square units is

$A$	$B$
$D$	$C$

- a. 175   b. 190   c. 180   d. 200   e. 185
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2.  $A$  and  $B$  together can do a job in 2 days.  $B$  and  $C$  can do it in 4 days; and  $A$  and  $C$  in 2.4 days. The number of days for  $A$  to do it alone is

- a. 5   b. 3   c. 2.75   d. 3.4   e. 4
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3. If  $k \neq 0$  and the parabola  $y = -x^2 + kx - 8k$  has its vertex on the  $x$ -axis, then  $k$  is

- a. 4   b. 8   c. 16   d. 32   e. 64
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4. If two poles 30 ft. and 50 ft. tall are 100 ft. apart, then the height in feet from ground level of the intersection of the lines joining the top of each pole to the foot of the opposite pole is

- a. 16.5   b. 18.75   c. 19.25   d. 20.5   e. 20.75
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5. If  $\frac{xy - y^2}{x^2 - y^2} = \frac{5}{2}$ , then  $\frac{x}{y}$  equals

- a.  $\frac{1}{2}$    b.  $\frac{5}{2}$    c.  $-\frac{5}{2}$    d.  $\frac{-2}{5}$    e.  $\frac{-3}{5}$
- 

6. If  $\begin{cases} a + b^2 + ab = 31 \\ b + a^2 + ab = 11 \end{cases}$ , and  $a$  and  $b$  are positive real numbers, then  $a + b$  is

- a. 6   b.  $\frac{3}{2}$    c. 8   d. 1   e. -1
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7. If  $(x^2 - 5x + 6)(3x^2 + mx + n) = 3x^4 - 17x^3 + 29x^2 - 17x + 6$ , then  $mn$  equals

- a. 10   b. -2   c. 18   d. -6   e. 8
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8. If  $x$  is a real number and  $\frac{2^{2x}}{4} - 2^{x-1} - 6 = 0$ , then  $x$  is
- a.  $\log\left(\frac{3}{2}\right)$    b.  $\log(2)\log(3)$    c.  $\frac{1}{2}\ln(3)$    d.  $\log_2(3)+1$    e.  $\log_2(3)$
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9. If the front wheel of a bicycle travels in a circle of radius 7.5 ft. while its rear wheel travels in a circle of radius 6 ft., then the distance between the axles of the bicycle in feet is
- a. 3.5   b. 4   c. 4.5   d. 3   e. 3.75
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10. A girls' camp is located 300 rods from a straight road  $A$ . A boys' camp is located 500 rods from the girls' camp on road  $A$ . A canteen on road  $A$  is equidistant to both camps. The distance of the canteen from each of the camps in rods is
- a. 375   b. 350.5   c. 325.5   d. 310   e. 312.5
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11. The square of a difference of two real numbers is 60, and the sum of their squares is 84. The absolute value of their sum is
- a.  $6\sqrt{3}$    b. 25   c.  $8\sqrt{2}$    d.  $4\sqrt{3}$    e.  $3\sqrt{5}$
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12. In a group of cows and chickens, the number of legs was 18 more than twice the number of heads. The number of cows in the group is
- a. 5   b. 7   c. 8   d. 9   e. 10
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13. The  $y$ -intercept of the line tangent to  $x^2 + y^2 = 1$  at  $\left(\frac{-5}{13}, \frac{12}{13}\right)$  is
- a.  $\frac{13}{12}$    b.  $\frac{7}{6}$    c.  $\frac{5}{4}$    d.  $\frac{4}{3}$    e.  $\frac{17}{12}$
- 
14. A car travels from  $A$  to  $B$  at 36 mph and then returns from  $B$  to  $A$  along the same path. If the roundtrip average speed was 48 mph, then the average speed of the return trip in mph was
- a. 64   b. 68   c. 72   d. 76   e. 80
- 
15. If  $(\log_k(x))(\log_4(k^2)) = 6$ , then  $x$  equals
- a. 16   b. 24   c. 36   d. 64   e. 128
- 
16. A circle is inscribed in a triangle with sides 8, 15, and 17. The radius of the inscribed circle is
- a. 2   b. 3   c. 4   d. 5   e. 6
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17. Two straight sticks are given of lengths 1 foot and 6 feet. If the 6 foot stick is cut into 2 pieces at a random location, then the probability that the resulting 3 sticks form the edges of a triangle is

- a.  $\frac{1}{3}$    b.  $\frac{1}{2}$    c.  $\frac{1}{6}$    d.  $\frac{1}{12}$    e.  $\frac{1}{4}$
- 

18. If  $(f \circ g)(x) = \frac{3x+6}{4x+23}$  and  $g(x) = \frac{x-1}{x+8}$ , then  $f(x)$  equals

- a.  $\frac{x-1}{x+1}$    b.  $\frac{x+2}{x+3}$    c.  $\frac{2x+1}{8x+3}$    d.  $\frac{2x+1}{x+3}$    e. n.o.t.
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19. A quarter is placed on a table. The number of quarters that can be replaced around it so that each is tangent to it and two others is

- a. 4   b. 5   c. 6   d. 8   e. 12
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20. If the digits  $a$  and  $b$  of a 2-digit (nonzero) decimal number  $x = ab$  are interchanged, the resulting 2-digit decimal number  $y = ba$  is 20% larger than  $x$ . The value of  $a + b^2$  is

- a. 29   b. 35   c. 12   d. 21   e. 18
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21. Two students were given the equation  $x^2 + bx + c = 0$  to solve. The first student inadvertently solved  $x^2 + bx + c_1 = 0$  and the second student solved  $x^2 + b_1x + c = 0$ . The solutions obtained by the first student were 2 and 8, and the solutions obtained by the second student were  $-1$  and  $-9$ . Then  $b^2 + c$  is

- a. 181   b. 109   c. 164   d. 144   e. 156
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22. The sum of 7 of 8 consecutive integers is 2018. The sum of all 8 integers is

- a. 2240   b. 2150   c. 2308   d. 2248   e. 2305
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23. Four distinct points are randomly chosen from the grid

$G = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$  in an  $xy$ -plane. The probability that they are vertices of a rectangle is

- a.  $\frac{1}{14}$    b.  $\frac{4}{21}$    c.  $\frac{1}{126}$    d.  $\frac{1}{21}$    e.  $\frac{5}{63}$
- 

24. Two chords of a circle intersect at right angles. The segments of one chord are 3 and 4 and the segments of the other chord are 6 and 2. The diameter of the circle is

- a. 7   b. 8   c. 12   d.  $\sqrt{61}$    e.  $\sqrt{65}$
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25. A 32 oz. tank is initially full of green colored water. It is turned into red colored water by repeatedly stirring the tank to a uniform mixture, and then draining 8 oz. of water from the tank and replacing it with 8 oz. of red colored water. If  $r_n$  is the number of ounces of red water in the tank at the completion of  $n$  repetitions of this process, then  $r_n$  is
- a.  $32(1-(3/4)^n)$    b.  $(32/3)(1-(1/4)^n)$    c.  $(128/15)(1-(1/4)^{n+1})$   
d.  $(128/23)(2-(3/4)^{n+1})$    e. n.o.t.
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26. Two baseball teams play a 7 game series. Assume each team has equal probability of winning any one of the games, and the series ends when either team wins 4 games. The probability that the series will end at the completion of the 6<sup>th</sup> game is
- a.  $\frac{5}{16}$    b.  $\frac{2}{3}$    c.  $\frac{6}{7}$    d.  $\frac{1}{4}$    e.  $\frac{3}{32}$
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27. The maximum value of  $y = |x-5| + |x+5|$  for  $|x+5| \leq .02$  is
- a. .19996   b. .1996   c. 1.996   d. 2.004   e. .2004
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28. If all angle measures are in degrees, then  $\cos(7)\cos(53) - \cos(8)\cos(38)$  equals
- a.  $\frac{1-\sqrt{3}}{4}$    b.  $\frac{1+\sqrt{3}}{4}$    c.  $\frac{1-\sqrt{3}}{2}$    d.  $\frac{1+\sqrt{3}}{2}$    e. n.o.t.
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29. The largest real number  $c$  such that for all real numbers  $x$ ,  $\cos(x) \geq \sin(x) + c$  is
- a. -1   b.  $-\frac{\sqrt{2}}{2}$    c.  $\frac{\sqrt{2}}{2}$    d.  $-\sqrt{2}$    e.  $\sqrt{2}$
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30. The remainder when  $4^{87} + 6^{87}$  is divided by 25 is
- a. 0   b. 5   c. 10   d. 20   e. n.o.t.
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**Tie Breaker**

Name \_\_\_\_\_

School Code \_\_\_\_\_

**Work the tie breaker problem on this page, tear off this sheet, and then hand it in along with your answer form. Write as much as you can. Partial credit will be given.**

**Tie Breaker:** The sum  $S_n$  of the first  $n$  terms of the sequence  $(a_n)$  of positive integers satisfies

$$a_n + \frac{1}{a_n} = 2S_n.$$

- a. Find  $a_1$
- b. Find  $a_2$
- c. Find  $a_3$
- d. Develop and prove a formula for  $a_n$ .